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Journal of Sound and Vibration 308 (2007) 692–698
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Characteristics of torsional vibrations of a shaft with unbalance

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Accepted 2 April 2007
The peer review of this article was organized by the Guest Editor
Available online 30 May 2007

Abstract

The aim of this paper is to study the characteristics of torsional vibration of a rotor with unbalance by numerical simulation. It can be concluded that synchronous torsional vibration accompanying with small higher harmonic components are excited, except when the speed of rotation is near, or equal to, half the natural frequency of torsional vibration, the bisynchronous component is much more remarkable than other harmonic components including the degraded synchronous one, and torsional vibration of the shaft can result in lateral vibration with bisynchronous frequency. Especially, when the rotating frequency is near the natural torsional frequency, where torsional vibration is strongest, the bisynchronous vibration becomes rather strong.
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1. Introduction

Unbalance is one of the most common troubles of shafts. Long time ago, the behavior of lateral vibration of a shaft with unbalance had been investigated. That is, unbalance causes synchronous lateral vibration. However, it can be known from the rotor dynamic equations that the torsional vibration is as an important part as the lateral one. Furthermore, the equations of motion show that lateral and torsional vibrations are coupled [1–3]. Only through studying the indispensable characteristics of torsional vibration besides the characteristics of lateral vibration, can we completely understand the characteristics of vibration of a rotor with unbalance.

Cohen and Porat [1] considered an unbalanced gyroscopic rotor, driven by a shaft rigid to bending and flexible to torsion. The nonlinear equations of motion are investigated by an asymptotic method. This approach does not yield a closed solution, but permits determination of the stability conditions at the combination resonances and gives an indication that in certain cases, superimposed damping causes considerable enlargement of the instability zones. Lateral vibration can induce torsional vibration and vice-versa. Bernasconi [2] reported that lateral vibration of a shaft with unbalance is able to induce torsional vibration whose frequency is double that of the rotational frequency when the speed of rotation is near, or equal to, half the natural frequency of torsional vibration. Nataraj [3] derived accurate expressions for the energy term and obtained the relevant equations and boundary conditions using a perturbation approach to show clearly the level and extent of interaction between torsion and flexure. He et al. [4] built a differential

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0022-460X/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2007.04.005

equations of coupled flexural-torsion vibration for a Jeffcott rotor model with unbalance mass. They used the small parameter perturbation method to analysis the characteristic of coupled vibrations. Fu et al. reported that when rotating speed of rotor met some special conditions, a kind of nonlinear vibraton phenomenon, which are different from the oil whirl even though there may be half-frequency whirl, might appear on an axisymmetrical rotor [5]. The aim of this paper is to research more roundly the characteristics of torsional vibration and the mutual influence between lateral and torsional vibration of a simple shaft with unbalance.

2. Dynamic equations

For an unbalanced Jeffcot rotor, as shown in Fig. 1, O' the rotating center, O the disk center, O_e the mass center, φ the angular displacement of mass, φ_0 the angular displacement of disk center, e the eccentricity of the disk, the dynamic equations of this shaft can be written as follows [1,2]:

$$\begin{cases} \ddot{X} + 2n_x\dot{X} + \omega_{nx}^2 X = e(\dot{\varphi}^2 \cos\varphi + \ddot{\varphi}\sin\varphi), \\ \ddot{Y} + 2n_y\dot{Y} + \omega_{ny}^2 Y = e(\dot{\varphi}^2 \sin\varphi - \ddot{\varphi}\cos\varphi) - g, \\ \ddot{\varphi} + 2n_\varphi\dot{\varphi} + \omega_{n\varphi}^2\varphi = em/J(\omega_{ny}^2 Y\cos\varphi - \omega_{nx}^2 X\sin\varphi), \end{cases} \tag{1}$$

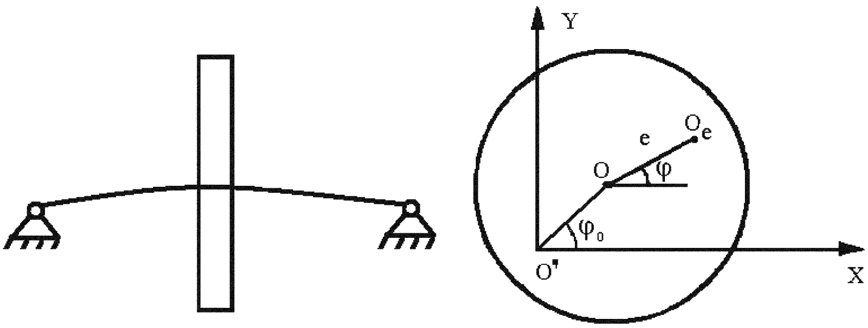

Fig. 1. Sketch map of a Jeffcott rotor with unbalance.

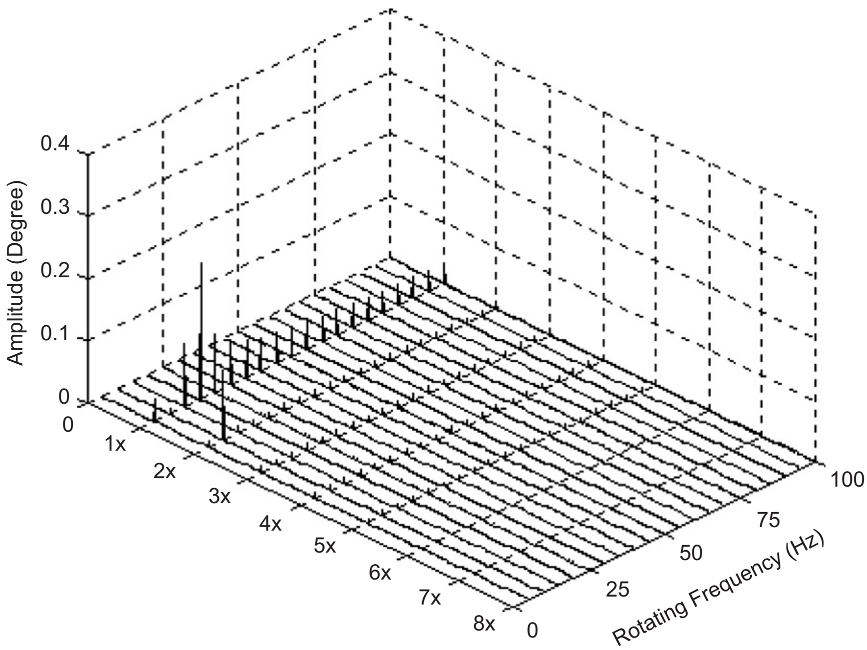

Fig. 2. Waterfall diagram of torsional vibration.

where, $(X,\ Y)$ is the displacements of the disk, φ the angular displacement, $\varphi = \Omega t + \Delta\theta$, Ω the frequency of rotation, t the time, $\Delta\theta$ the torsional vibrational angle, m the mass of the disk, g the acceleration of gravity, J the rotating inertia of the disk with respect to the mass center. n_x, n_y and n_φ are damping coefficients of X, Y directional lateral vibrations and torsional vibration, ω_{nx} and ω_{ny} are the lateral natural frequencies in X and Y direction, respectively and $\omega_{n\varphi}$ the torsional natural frequencies of the shaft.

3. Results of simulations and comparing with experiment in reference

The algorithm of Wilson-θ is used to solve Eq. (1). When $t = 0$, the initial conditions for Eq. (1) can be given easily as follows:

$$\begin{cases} X(0) = 0, \\ \dot{X}(0) = 0, \end{cases} \quad \begin{cases} Y(0) = -g/\omega_{ny}, \\ \dot{Y}(0) = 0, \end{cases} \quad \begin{cases} \varphi(0) = -\pi/2, \\ \dot{\varphi}(0) = 0. \end{cases} \tag{2}$$

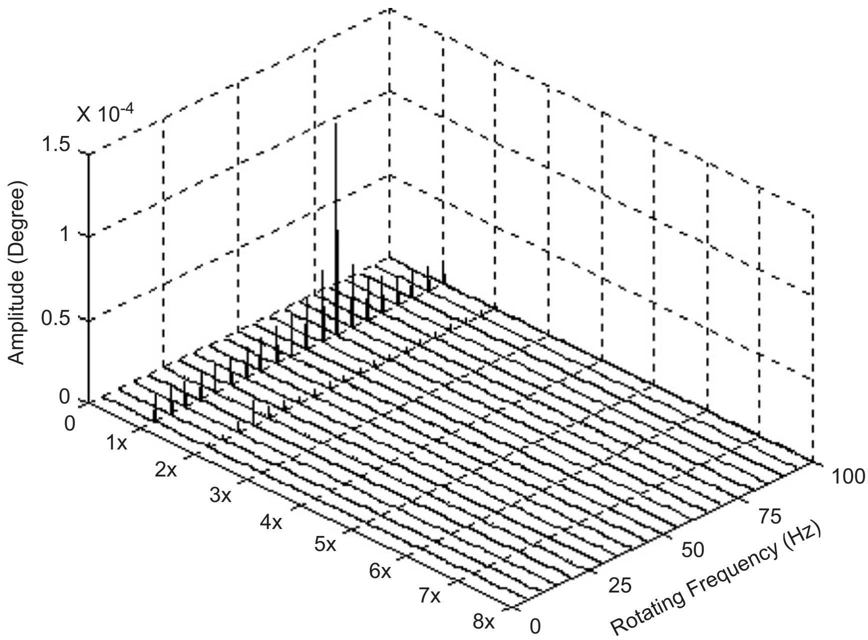

Fig. 3. Waterfall diagram of lateral vibration in the x direction.

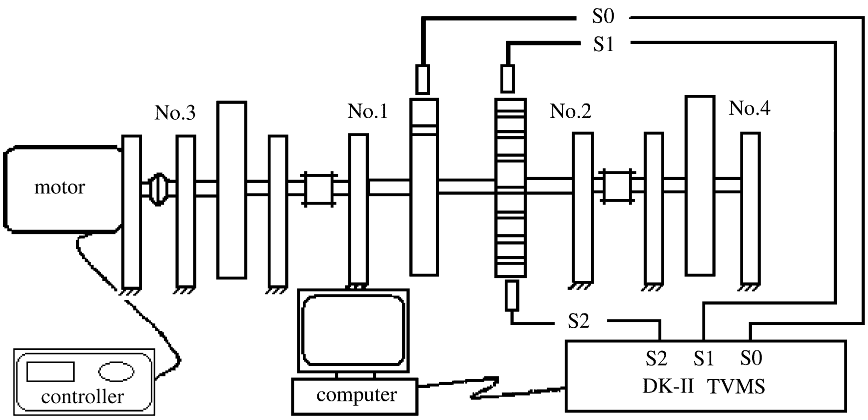

Fig. 4. Experimental bench.

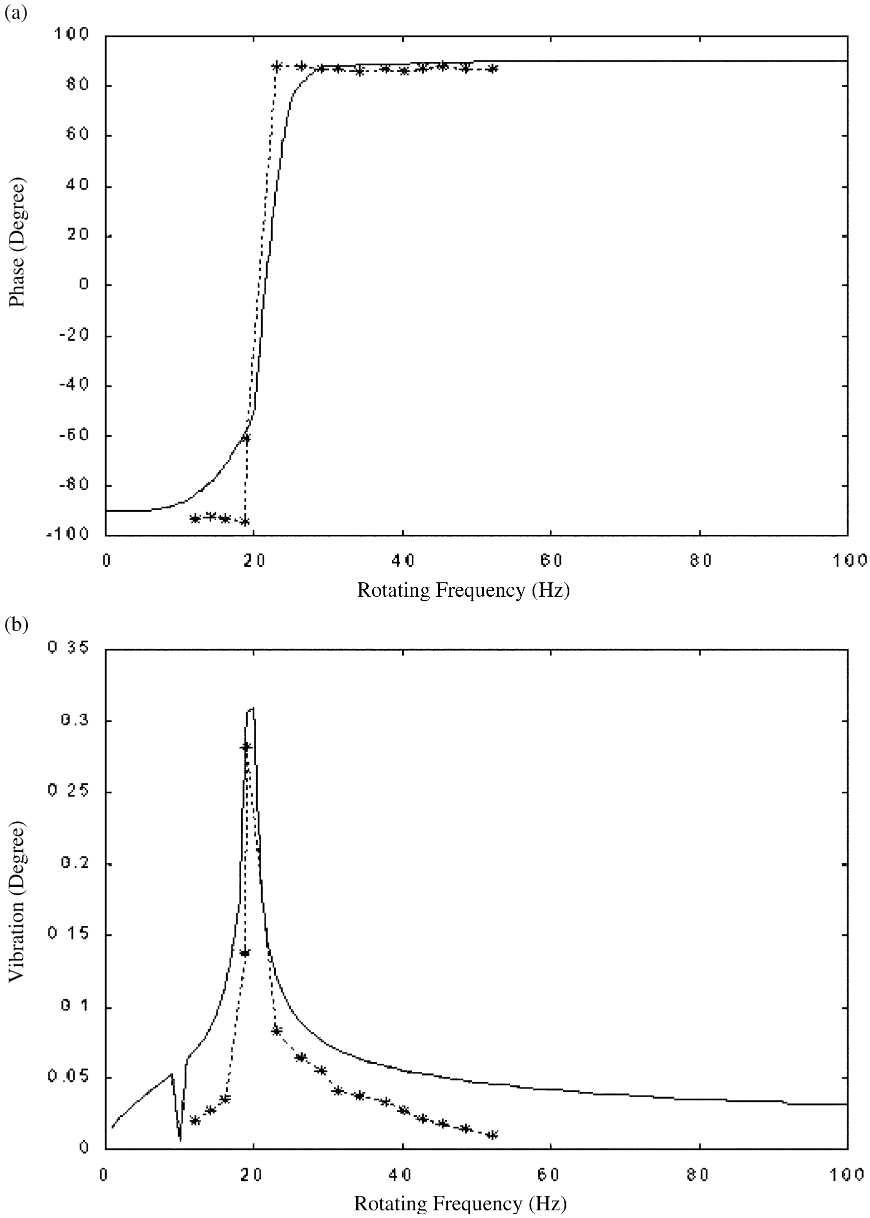

Fig. 5. Comparison of bode diagrams between simulation and experiment for torsional vibration (a) phase of synchronous vibration; (b) amplitude synchronous vibration. ∗- - - ∗- - - ∗, experiment; ———, simulation.

The dynamic response of a shaft with unbalance, which has been tested in Ref. [7], is simulated in this paper. The lateral natural frequencies in X and Y direction are equal. The torsional natural frequency $\omega_{n\varphi}$ of this shaft is 19 Hz and lateral natural frequencies ω_{nx} and ω_{ny} are 65 Hz. The ratio of $n_x/\omega_{nx} = n_y/\omega_{ny} = n_\varphi/\omega_{n\varphi} = 0.25$, $e = 7.0 \times 10^{-5}$ m. Fig. 2 gives the waterfall diagram of torsional vibration. The figure shows clearly that synchronous torsional vibration accompanying with small higher harmonic components is excited, except when the speed of rotation is near, or equal to, half the natural frequency of torsional vibration, the bisynchronous component is much more remarkable than other harmonic components including the degraded synchronous one. Fortunately, the phenomenon of bisynchronous torsional vibration and the corresponding speed region have been reported and experimentally verified by Bernasconi [2] in his shaft. Although the shafts used in Bernasconi's experiment and this simulation are quite different, the characteristics of torsional vibration excited by unbalance are identical.

Fig. 3 gives the waterfall diagram of lateral vibration. The synchronous vibration is well known and not worth mentioning here. However, the noteworthy point in this figure is that bisynchronous lateral vibration is induced. Especially, when the rotating frequency is near the natural torsional frequency, where torsional vibration is strongest, the bisynchronous vibration becomes as strong as the synchronous one.

The measurement system of lateral and torsional vibration, which is called DK-II TVMS, has been introduced in Refs. [6,7] by Fu and Huang. The principle of measurement system of lateral and torsional vibration is based on the time interval measurement of pulse signals. By setting a particular tooth on a rotor

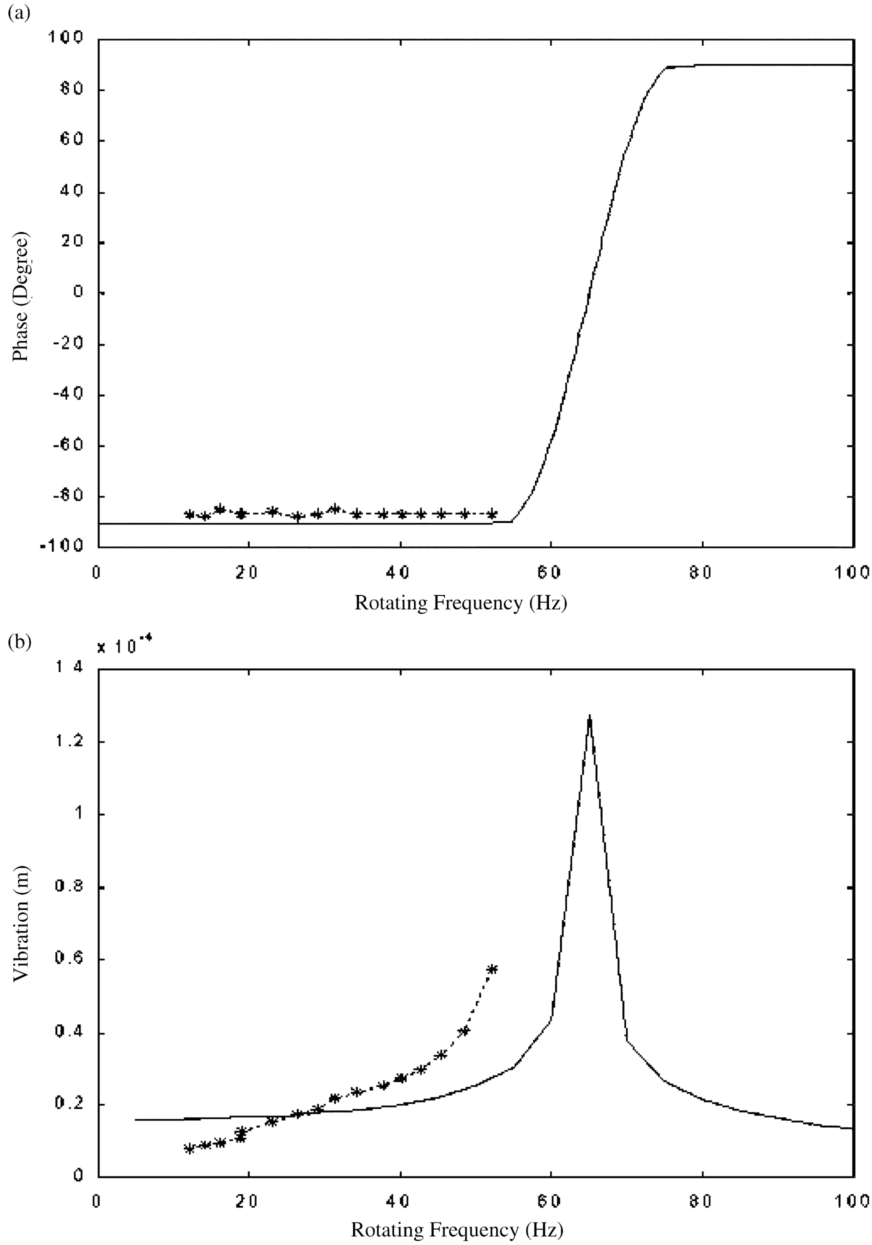

Fig. 6. Comparision of bode diagrams between simulation and experiment for lateral vibration (a) phase of synchronous vibration; (b) amplitude synchronous vibration. ∗ - - - ∗ - - - ∗, experiment; ———, simulation.

disk to indicate the starting point of each rotating period, we can acquire the time series of every tooth arrival. The time interval between the teeth is the superposition of ωt, torsional vibration and lateral vibration in the direction which is vertical to the sensor. By resolving the two pulse signals which are acquired from the sensors installed on both sides in the horizontal plane through the shaft into torsional and lateral vibration, this system can distinguish an angle of torsional vibration of 0.0018° as well as accurate lateral vibration. It is inherently insensitive to unequal distribution of the teeth and can be installed conveniently [6,7].

The experimental shaft as shown in Fig. 4 has been introduced in Ref. [8] by Huang. Disk No. 1 is of one tooth that can generate one pulse signal in one rotating period. The signal picked by the sensor which is located against this disk is sent to S0 channel of the DK-II TVMS. Disk No. 2 has 16 teeth with two sensors are installed on both sides in the horizontal plane through the shaft, and sent to S1 and S2 channel of the DK-II TVMS, respectively. For some reasons, the frequency of rotation is confined in the range of 12–54 Hz. The first torsional and lateral natural frequencies of this shaft are 19 and 65 Hz, respectively.

Fig. 5 gives the comparison of Bode diagrams between simulation and experiment reported for torsional vibration at synchronous frequency. It shows that the results are fairly consistent in the range of experiment. Bernasconi [2] gave his experiments in the region of half-natural frequency. The result of obvious bisynchronous torsional vibration accompanying with inapparent synchronous vibration given by him is consistent with that from this simulation in Fig. 2. Fig. 5 also shows that when the rotor passes critical speed of torsional vibration, the phase of synchronous vibration will reverse. Except in the region of half-natural frequency, the behaviors of amplitude and phase of synchronous torsional vibration are similar with that of lateral vibration which have been well known. When the speed is near, or equal to, half natural frequency, synchronous torsional vibration is obviously degraded. For the damping coefficients existed in experiment and used in simulation may be different, the difference of amplitudes from simulation and experiment is reasonable. However, this difference is not an obstacle for us to understand the characteristics of torsional vibration.

Fig. 6 gives the comparison of Bode diagrams between simulation and experiment [8] for lateral vibration at synchronous frequency. We cannot find anything new from the numerical simulation of synchronous lateral vibration. In other words, torsional vibration does not induce any change in synchronous lateral vibration.

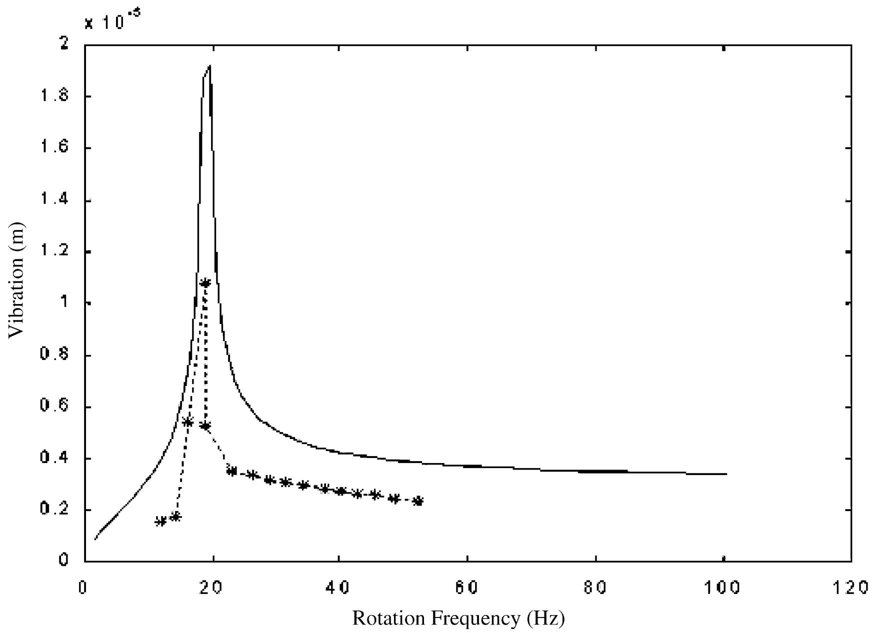

Fig. 7. Comparison of 2× amplitude between simulation and experiment. *---*---*, experiment; ———, simulation.

Fig. 7 gives the comparison of bisynchronous amplitude between simulation and experiment for lateral vibration. The coincident results show that torsional vibration excites bisynchronous lateral vibration. At the frequency of torsional resonance, bisynchronous lateral vibration is the strongest.

4. Conclusion

The characteristics of torsional vibration of a rotor with unbalance are carefully studied in this paper. It can be concluded that synchronous torsional vibration accompanying with small higher harmonic components are excited, except when the speed of rotation is near, or equal to, half the natural frequency of torsional vibration, the bisynchronous component is much more remarkable than other harmonic components including the degraded synchronous one, and torsional vibration of the shaft can result in lateral vibration with bisynchronous frequency. Especially, when the rotating frequency is near the natural torsional frequency, the bisynchronous vibration becomes as strong as the synchronous one.

Acknowledgments

This work was supported by the National Natural Science Foundation of China Grant No. 50576049 and Specialized Research Fund for the Doctoral Program of Higher Education (20060280017). It was also supported partly by Shanghai Leading Academic Discipline Project Grant No. Y0103 and supported by Shanghai Science and Technology Committee Grant No. 05JC14066 partly.

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